# Rotor Spectra, Berry Phases, and Monopole Fields: from Antiferromagnets to QCD

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The order parameter of a finite system with a spontaneously broken continuous global symmetry acts as a quantum mechanical rotor. Both antiferromagnets with a spontaneously broken $SU(2)_s$ spin symmetry and massless QCD with a broken $SU(2)_L \times SU(2)_R$ chiral symmetry have rotor spectra when considered in a finite volume. When an electron or hole is doped into an antiferromagnet or when a nucleon is propagating through the QCD vacuum, a Berry phase arises from a monopole field and the angular momentum of the rotor is quantized in half-integer units.

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Berry phases and monopole fields are familiar from adiabatic processes in quantum mechanical systems [1, 2]. For example, the slow rotation of the nuclei in a diatomic molecule is influenced by a geometric vector potential generated by the fast motion of the electrons [3, 4]. The Abelian and non-Abelian monopole content of these vector potentials was worked out elegantly by Moody, Shapere, and Wilczek [5]. In this paper we discuss Berry phases and monopole fields for rotors arising in condensed matter and particle physics systems with a spontaneously broken continuous global symmetry.

The undoped precursors of layered cuprate high-temperature superconductors are antiferromagnets with a spontaneously broken $SU(2)_s$ spin symmetry. When one considers an antiferromagnet of finite volume $V$ at very low temperatures, the dynamics are dominated by the spatially independent zero-mode of the staggered magnetization order parameter

$$\vec{e}(t) = (\sin\theta(t)\cos\phi(t), \sin\theta(t)\sin\phi(t), \cos\theta(t)), \quad (1)$$

which represents a slow quantum mechanical rotor governed by the Lagrangian [6]

$$\mathcal{L} = \frac{\Theta}{2}\partial_t\vec{e}\cdot\partial_t\vec{e} = \frac{\Theta}{2}\left[(\partial_t\theta)^2 + \sin^2\theta(\partial_t\varphi)^2\right]. \quad (2)$$

Integrating out the fast non-zero modes of the staggered magnetization at one loop, and assuming a 2-dimensional quadratic periodic volume, the moment of inertia was determined by Hasenfratz and Niedermayer [6] as

$$\Theta = \frac{\rho_s V}{c^2}\left[1 + \frac{3.900265}{4\pi}\left(\frac{c}{\rho_s L}\right) + \mathcal{O}\left(\frac{1}{L^2}\right)\right], \quad (3)$$

where $\rho_s$ is the spin stiffness and $c$ is the spinwave velocity. The momenta conjugate to $\theta$ and $\varphi$ are

$$p_\theta = \frac{\delta\mathcal{L}}{\delta\partial_t\theta} = \Theta\ \partial_t\theta,\ p_\varphi = \frac{\delta\mathcal{L}}{\delta\partial_t\varphi} = \Theta\ \sin^2\theta\ \partial_t\varphi, \quad (4)$$

and the resulting Hamiltonian

$$H = -\frac{1}{2\Theta}\left(\frac{1}{\sin\theta}\partial_\theta[\ \sin\theta\partial_\theta] + \frac{1}{\sin^2\theta}\partial_\varphi^2\right) = \frac{\vec{L}^2}{2\Theta} \quad (5)$$

is just the Laplacian on the sphere $S^2$. Correspondingly, the energy spectrum is that of a quantum mechanical rotor with angular momentum $l \in \{0, 1, 2, ...\}$, i.e.

$$E_l = \frac{l(l+1)}{2\Theta}, \quad (6)$$

with each state being $(2l+1)$-fold degenerate. The rotor features have been verified in numerical simulations of the antiferromagnetic quantum Heisenberg model [7, 8] (or equivalently of the $t$-$J$ model at half-filling). It should be noted that a quantum ferromagnet does not behave as a rotor because its order parameter — the uniform magnetization — is a conserved quantity.

When a single hole or electron is doped into the antiferromagnet, the spin of the system changes by 1/2 and thus the angular momentum of the resulting rotor must then be quantized in half-integer units. As we will see, in the language of low-energy effective theories, this half-integer quantization is a result of Berry phases and monopole fields. Systematic low-energy effective theories for charge carriers in an antiferromagnet were recently constructed in [9, 10]. The leading terms in the low-energy Lagrangian of holes or electrons with a small momentum $\vec{p}$ are given by

$$\mathcal{L} = \frac{\Theta}{2}\partial_t\vec{e}\cdot\partial_t\vec{e} + \Psi^\dagger\left[E(\vec{p}) - i\partial_t + v_t^3\sigma_3 + \lambda V_t\right]\Psi. \quad (7)$$

Here $\Psi(t) = \begin{pmatrix}\psi_+(t)\\ \psi_-(t)\end{pmatrix}$ is a two-component Grassmann valued field describing fermions with spin parallel (+) or anti-parallel (−) to the local staggered magnetization. It should be noted that we have suppressed an additional flavor index of the hole fields in a doped cuprate antiferromagnet [10], which distinguishes between holes from different pockets in the Brillouin zone. The fermion energy $E(\vec{p})$ as well as $\lambda$ can be determined by integrating out the non-zero momentum modes of the staggered magnetization, e.g. at one loop. For hole- or electron-doped cuprates as well as for the $t$-$J$ model it was predicted

that $\lambda = 0$ [10], while for other antiferromagnets in general $\lambda \neq 0$ [9]. The Abelian vector potential $v_t^3(t)$ is the diagonal component of the composite vector field

$$v_t = u\partial_t u^\dagger = i v_t^a \sigma_a = i v_t^3 \sigma_3 + i V_t. \tag{8}$$

Here $\sigma_a$ are the Pauli matrices and

$$u = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2}\exp(-i\varphi) \\ -\sin\frac{\theta}{2}\exp(i\varphi) & \cos\frac{\theta}{2} \end{pmatrix} \tag{9}$$

represents a transformation which rotates $\vec{e}(t)$ into the 3-direction. One then obtains

$$\begin{aligned} v_t^3 &= \sin^2\frac{\theta}{2}\partial_t\varphi, \\ V_t &= \frac{1}{2}\sin\theta(\cos\varphi\ \sigma_1 + \sin\varphi\ \sigma_2)\ \partial_t\varphi \\ &+ \frac{1}{2}(\ \sin\varphi\ \sigma_1 - \cos\varphi\ \sigma_2)\ \partial_t\theta. \end{aligned} \tag{10}$$

These velocity-dependent terms give rise to a modification of the canonically conjugate momenta such that

$$\Theta\ \partial_t\theta = p_\theta + iA_\theta,\ \Theta\ \partial_t\varphi = \frac{1}{\sin^2\theta}(p_\varphi + iA_\varphi), \tag{11}$$

with the non-Abelian vector potential

$$\begin{aligned} A_\theta &= i\frac{\lambda}{2}(\sin\varphi\ \sigma_1 - \cos\varphi\ \sigma_2), \\ A_\varphi &= i\sin^2\frac{\theta}{2}\sigma_3 + i\frac{\lambda}{2}\sin\theta(\cos\varphi\ \sigma_1 + \sin\varphi\ \sigma_2), \end{aligned} \tag{12}$$

and the corresponding field strength

$$F_{\theta\varphi} = \partial_\theta A_\varphi - \partial_\varphi A_\theta + [A_\theta, A_\varphi] = i\frac{1-\lambda^2}{2}\sin\theta\ \sigma_3. \tag{13}$$

Remarkably, the resulting geometric Berry gauge field is exactly the same as for a diatomic molecule [5]. For cuprates ($\lambda = 0$) the vector potential is Abelian and describes a monopole with quantized magnetic flux. The path $\vec{e}(t)$ in periodic Euclidean time defines a closed loop $\mathcal{C}$ on $S^2$. The Boltzmann factor in the path integral contains a Wilson loop along $\mathcal{C}$ which manifests itself as a Berry phase. Using Stokes' theorem, the Berry phase is given by the magnetic flux enclosed in $\mathcal{C}$. Since the enclosed flux is well-defined only up to the area $4\pi$ of $S^2$, the magnetic charge is $\pm\frac{1}{2}$ as a consequence of the Dirac quantization condition. For a general antiferromagnet ($\lambda \neq 0$) the vector potential becomes non-Abelian and the flux is no longer quantized.

The resulting Hamilton operator takes the form

$$\begin{aligned} H(\lambda) &= -\frac{1}{2\Theta}\left\{\frac{1}{\sin\theta}(\partial_\theta - A_\theta)[\ \sin\theta(\partial_\theta - A_\theta)]\right. \\ &+ \left.\frac{1}{\sin^2\theta}(\partial_\varphi - A_\varphi)^2\right\} + E(\vec{p}). \end{aligned} \tag{14}$$

The solution for the energy spectrum can be obtained along the lines of [5]. First, one can show that the Hamiltonian $H(0)$ (with $\lambda = 0$) commutes with the angular momentum operators

$$\begin{aligned} J_\pm &= \exp(\pm i\varphi)\left(\pm\ \partial_\theta + i\cot\theta\ \partial_\varphi - \frac{1}{2}\tan\frac{\theta}{2}\sigma_3\right), \\ J_3 &= -i\partial_\varphi - \frac{\sigma_3}{2}, \end{aligned} \tag{15}$$

and is given by

$$H(0) = \frac{1}{2\Theta}\left(\vec{J}^2 - \frac{1}{4}\right) + E(\vec{p}), \tag{16}$$

such that the energy spectrum takes the form

$$E_j(0) = \frac{1}{2\Theta}\left[j(j+1) - \frac{1}{4}\right] + E(\vec{p}). \tag{17}$$

Here $j$ is a half-integer. In this case, each state is $2(2j+1)$-fold degenerate because the fermion sectors $+$ and $-$ cost the same energy. The corresponding wave functions with half-integer angular momentum are monopole harmonics [11, 12]. In particular, the ground state wave functions are

$$\begin{aligned} Y^\pm_{\frac{1}{2},\pm\frac{1}{2}}(\theta,\varphi) &= \frac{1}{\sqrt{2\pi}}\sin\frac{\theta}{2}\exp(\pm i\varphi), \\ Y^\pm_{\frac{1}{2},\mp\frac{1}{2}}(\theta,\varphi) &= \frac{1}{\sqrt{2\pi}}\cos\frac{\theta}{2}. \end{aligned} \tag{18}$$

It should be noted that $Y^+_{\frac{1}{2},\frac{1}{2}}(\theta,\varphi)$ and $Y^-_{\frac{1}{2},-\frac{1}{2}}(\theta,\varphi)$ have coordinate singularities at $\theta = \pi$ related to the Dirac string. Following Wu and Yang [12], one can avoid the coordinate singularity by introducing different coordinate patches glued together by gauge transformations.

The Hamiltonian with $\lambda \neq 0$ takes the form

$$H(\lambda) = H(0) + \frac{1}{2\Theta}\left(\lambda C + \frac{1}{2}\lambda^2\right), \tag{19}$$

and still commutes with $\vec{J}$ of eq.(15). Here

$$\begin{aligned} C &= -i\left(\sin\varphi\partial_\theta + \frac{\cos\varphi}{\sin\theta}\partial_\varphi - \frac{1}{2}\sin\varphi\tan\frac{\theta}{2}\right)\sigma_1 \\ &+ i\left(\cos\varphi\partial_\theta - \frac{\sin\varphi}{\sin\theta}\partial_\varphi - \frac{1}{2}\cos\varphi\tan\frac{\theta}{2}\right)\sigma_2, \end{aligned} \tag{20}$$

and $[C, \vec{J}] = 0$. Using $C^2 = \vec{J}^2 + \frac{1}{4}$ one obtains the energy spectrum

$$E_j(\lambda) = \frac{1}{2\Theta}\left[j'(j'+1) + \frac{\lambda^2 - 1}{4}\right] + E(\vec{p}), \tag{21}$$

with $j' = j \pm \frac{\lambda}{2}$ and $j$ again being a half-integer. For $\lambda \neq 0$ the fermion sectors $+$ and $-$ get mixed and the previously degenerate $2(2j+1)$ states are now split into

two groups of $2j+1$ degenerate states. Interestingly, for $\lambda = \pm 1$ the monopole field strength of eq.(13) vanishes and $E_j(\pm 1) = \frac{1}{2\Theta} j'(j'+1)$ with $j' = j \pm \frac{1}{2}$. In that case, the rotor spectrum looks like the one of eq.(6) although the angular momentum $j$ is now a half-integer.

Let us now consider QCD with two massless flavors and thus with a spontaneously broken $SU(2)_L \times SU(2)_R$ chiral symmetry. When the theory is put in a finite spatial volume $V$, as it is the case in numerical simulations of lattice QCD, the chiral order parameter $U(t) \in SU(2)$ describes a quantum rotor with the Lagrangian

$$\mathcal{L} = \frac{\Theta}{4} \mathrm{Tr} \left[ \partial_t U^\dagger \partial_t U \right]. \tag{22}$$

At tree-level the moment of inertia is given by $\Theta = F_\pi^2 V$ where $F_\pi$ is the pion decay constant. The corresponding Hamiltonian is the Laplacian on the sphere $S^3$. The QCD rotor spectrum has been derived by Leutwyler [13] in the $\delta$-expansion of chiral perturbation theory [14] as

$$E_l = \frac{j_L(j_L+1) + j_R(j_R+1)}{\Theta} = \frac{l(l+2)}{2\Theta}. \tag{23}$$

In this case, $j_L = j_R$ with $l = j_L + j_R \in \{0, 1, 2, ...\}$ and each state is $(2j_L+1)(2j_R+1) = (l+1)^2$-fold degenerate. The low-energy dynamics of nucleons and pions is described by baryon chiral perturbation theory [15, 16, 17, 18]. When a nucleon with small momentum $\vec{p} = |\vec{p}|\vec{e}_p$ is propagating in the finite volume the Lagrangian reads

$$\begin{aligned} \mathcal{L} &= \frac{\Theta}{4} \mathrm{Tr} \left[ \partial_t U^\dagger \partial_t U \right] \\ &+ \Psi^\dagger \left[ E(\vec{p}) - i\partial_t - i v_t - i\lambda(\vec{\sigma} \cdot \vec{e}_p) a_t \right] \Psi. \end{aligned} \tag{24}$$

Here $\Psi(t)$ is a Pauli spinor with a flavor index distinguishing protons and neutrons and $\frac{\vec{\sigma}}{2}$ is the nucleon spin. At tree level $E(\vec{p}) = M + \vec{p}^2/2M$ and $\lambda = g_A|\vec{p}|/M$, where $M$ is the mass and $g_A$ is the axial vector coupling of the nucleon. As for the antiferromagnet, the parameters $\Theta$, $E(\vec{p})$, and $\lambda$ get renormalized by the coupling to non-zero momentum pion modes. In the QCD case $u^2 = U$ and

$$v_t = \frac{1}{2} \left( u \partial_t u^\dagger + u^\dagger \partial_t u \right), \ a_t = \frac{1}{2i} \left( u \partial_t u^\dagger - u^\dagger \partial_t u \right). \tag{25}$$

Parameterizing

$$\begin{aligned} &U(t) = \cos\alpha(t) + i \sin\alpha(t) \vec{e}_\alpha(t) \cdot \vec{\tau}, \\ &\vec{e}_\alpha(t) = (\sin\theta(t)\cos\varphi(t), \sin\theta(t)\sin\varphi(t), \cos\theta(t)), \\ &\vec{e}_\theta(t) = (\cos\theta(t)\cos\varphi(t), \cos\theta(t)\sin\varphi(t), -\sin\theta(t)), \\ &\vec{e}_\varphi(t) = (-\sin\varphi(t), \cos\varphi(t), 0), \end{aligned} \tag{26}$$

one obtains

$$\begin{aligned} v_t &= i \sin^2 \frac{\alpha}{2} \left( \partial_t \theta \ \vec{e}_\varphi - \sin\theta \ \partial_t \varphi \ \vec{e}_\theta \right) \cdot \vec{\tau}, \\ a_t &= \left( \frac{\partial_t \alpha}{2} \vec{e}_\alpha + \sin\alpha \frac{\partial_t \theta}{2} \vec{e}_\theta + \sin\alpha \sin\theta \frac{\partial_t \varphi}{2} \vec{e}_\varphi \right) \cdot \vec{\tau}. \end{aligned} \tag{27}$$

Here $\vec{\tau}$ are the Pauli matrices for isospin.

The resulting Hamilton operator takes the form

$$\begin{aligned} H(\lambda) = &-\frac{1}{2\Theta} \left\{ \frac{1}{\sin^2\alpha} (\partial_\alpha - A_\alpha)[\ \sin^2\alpha (\partial_\alpha - A_\alpha)] \right. \\ &+ \frac{1}{\sin^2\alpha \sin\theta} (\partial_\theta - A_\theta)[\ \sin\theta (\partial_\theta - A_\theta)] \\ &\left. + \frac{1}{\sin^2\alpha \sin^2\theta} (\partial_\varphi - A_\varphi)^2 \right\} + E(\vec{p}), \end{aligned} \tag{28}$$

with the non-Abelian vector potential

$$\begin{aligned} A_\alpha &= i \frac{\lambda}{2} (\vec{\sigma} \cdot \vec{e}_p) \vec{e}_\alpha \cdot \vec{\tau}, \\ A_\theta &= i \left( \sin^2 \frac{\alpha}{2} \ \vec{e}_\varphi + \frac{\lambda}{2} (\vec{\sigma} \cdot \vec{e}_p) \sin\alpha \ \vec{e}_\theta \right) \cdot \vec{\tau}, \\ A_\varphi &= i \left( -\sin^2 \frac{\alpha}{2} \sin\theta \ \vec{e}_\theta \right. \\ &+ \left. \frac{\lambda}{2} (\vec{\sigma} \cdot \vec{e}_p) \sin\alpha \sin\theta \ \vec{e}_\varphi \right) \cdot \vec{\tau}, \end{aligned} \tag{29}$$

and the corresponding field strength

$$\begin{aligned} F_{\alpha\theta} &= i \frac{1-\lambda^2}{2} \sin\alpha \ \vec{e}_\varphi \cdot \vec{\tau}, \\ F_{\theta\varphi} &= i \frac{1-\lambda^2}{2} \sin^2\alpha \ \sin\theta \ \vec{e}_\alpha \cdot \vec{\tau}, \\ F_{\varphi\alpha} &= i \frac{1-\lambda^2}{2} \sin\alpha \ \sin\theta \ \vec{e}_\theta \cdot \vec{\tau}. \end{aligned} \tag{30}$$

This Berry gauge field is a non-Abelian analog of the Abelian monopole field we encountered for the antiferromagnet. The non-Abelian gauge field again has a coordinate singularity, in this case at $\alpha = \pi$, which corresponds to a Dirac string going through the south pole of $S^3$. This indicates that there is a non-Abelian magnetic monopole at the center of $S^3$.

The generators of $SU(2)_L \otimes SU(2)_R$ take the form

$$\begin{aligned} \vec{J}_L &= \frac{1}{2}(\vec{J} - \vec{K}), \ \vec{J}_R = \frac{1}{2}(\vec{J} + \vec{K}), \\ J_\pm &= \exp(\pm i\varphi) \left( \pm \ \partial_\theta + i \cot\theta \ \partial_\varphi \right) + \frac{\tau_\pm}{2}, \\ J_3 &= -i\partial_\varphi + \frac{\tau_3}{2}, \\ K_\pm &= \exp(\pm i\varphi) \left( i \sin\theta \ \partial_\alpha + i \cot\alpha \cos\theta \ \partial_\theta \mp \frac{\cot\alpha}{\sin\theta} \partial_\varphi \right. \\ &\mp \left. \frac{i}{2} \tan\frac{\alpha}{2} \ \vec{e}_\theta \cdot \vec{\tau} + \frac{1}{2} \tan\frac{\alpha}{2} \cos\theta \ \vec{e}_\varphi \cdot \vec{\tau} \right), \\ K_3 &= i \left( \cos\theta \ \partial_\alpha - \cot\alpha \sin\theta \ \partial_\theta \right) \\ &- \frac{1}{2} \tan\frac{\alpha}{2} \sin\theta \ \vec{e}_\varphi \cdot \vec{\tau}. \end{aligned} \tag{31}$$

The Hamiltonian $H(0)$ (with $\lambda = 0$) can be written as

$$H(0) = \frac{1}{2\Theta} \left( \vec{J}^{\,2} + \vec{K}^{\,2} - \frac{3}{4} \right) + E(\vec{p}), \tag{32}$$

such that the energy spectrum takes the form

$$E_j(0) = \frac{1}{2\Theta}\left[j(j+2) - \frac{1}{2}\right] + E(\vec{p}). \qquad (33)$$

In this case, $j_L = j_R \pm \frac{1}{2}$ and $j = j_L + j_R \in \{\frac{1}{2}, \frac{3}{2}, ...\}$. Each state is $2(j+\frac{1}{2})(j+\frac{3}{2})$-fold degenerate because the states with spin up and spin down cost the same energy.

The Hamiltonian with $\lambda \neq 0$ can be written as

$$H(\lambda) = H(0) + \frac{1}{2\Theta}\left(\lambda C + \frac{3}{4}\lambda^2\right), \qquad (34)$$

and it still commutes with $\vec{J}$ and $\vec{K}$. Here

$$\begin{aligned} C &= i(\vec{\sigma}\cdot\vec{e}_p)\left(\vec{e}_\alpha\partial_\alpha + \frac{1}{\sin\theta}\vec{e}_\theta\partial_\theta\right. \\ &+ \left.\frac{1}{\sin\alpha\sin\theta}\vec{e}_\varphi\partial_\varphi - \tan\frac{\alpha}{2}\vec{e}_\alpha\right)\cdot\vec{\tau}, \qquad (35)\end{aligned}$$

and $[C, \vec{J}] = [C, \vec{K}] = 0$. Using $C^2 = \vec{J}^{\,2} + \vec{K}^{\,2} + \frac{3}{4}$ one finally obtains the energy spectrum

$$E_j(\lambda) = \frac{1}{2\Theta}\left[j'(j'+2) + \frac{\lambda^2 - 1}{2}\right] + E(\vec{p}), \qquad (36)$$

with $j' = j \pm \frac{\lambda}{2}$, where $\pm$ refers to the spin eigenstates of $\vec{\sigma}\cdot\vec{e}_p$ with eigenvalues $\pm 1$. Thus we see that for $\lambda \neq 0$ the degeneracy is partly lifted and there are now two groups of $(j+\frac{1}{2})(j+\frac{3}{2})$-fold degenerate states. Remarkably, for $\lambda = \pm 1$ the non-Abelian field strength of eq.(30) vanishes and $E_j(\pm 1) = \frac{1}{2\Theta}j'(j'+2)$ with $j' = j \pm \frac{1}{2}$. Just as for an antiferromagnet with $\lambda = \pm 1$, the QCD rotor spectrum then looks like the one of eq.(23) although the system now has fermion number one.

The present study in the $\delta$-regime complements other investigations of finite volume effects in the one-nucleon sector of QCD in the $p$- [19, 20, 21], $\epsilon$- [22], and $\epsilon'$-regimes [23] of chiral perturbation theory. A comparison of numerical lattice QCD data with finite volume predictions of chiral perturbation theory may lead to an accurate determination of low-energy parameters including $F_\pi$ and some of the Gasser-Leutwyler coefficients. Before one can do this in the $\delta$-regime, one must match the volume-dependent parameters $\Theta$, $E(\vec{p})$, and $\lambda$ of the effective quantum mechanics to those of the infinite volume effective theory.

While it is difficult to simulate QCD in the chiral limit, simulations of a single hole in the $t$-$J$ model with an exact $SU(2)_s$ spin symmetry are possible using efficient cluster algorithms. Again, before one can extract the parameters of the systematic effective theory of magnons and holes [10] from a comparison with numerical data, it will be necessary to match the volume-dependent parameters $\lambda$ and $E(\vec{p})$ of the effective quantum mechanics to those of the infinite volume effective theory, e.g. at one loop. However, even without performing this matching calculation, one can check if indeed $\lambda = 0$, as predicted for the $t$-$J$ model in [10].

The effects discussed here are not limited to antiferromagnets or QCD, but arise for any finite system with a spontaneously broken continuous global symmetry (unless the order parameter is conserved). When a symmetry group $G$ breaks spontaneously to a subgroup $H$, the corresponding order parameter takes values in the coset space $G/H$. In a finite volume the symmetry is restored by a slow rotation of the order parameter. When a single fermion is added to the system, one expects Berry phases resulting from monopole gauge fields residing on the manifold $G/H$, with characteristic effects on the rotor spectrum. It would be interesting to work out these effects for general $G$ and $H$ and for an arbitrary fermion representation. A non-trivial case of physical interest is QCD with three massless flavors for which $G/H = SU(3)$ and the baryons transform as flavor octets.

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